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A new mathematical model of the dynamic of psychotherapy

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Abstract

The aim of the present study is to construct a mathematical model skilled in creating hard estimates about the dynamic of psychotherapy with a purpose of using it for better sequence therapists. The outcome of the calculations envisages a new viewpoint on the therapeutic relationship and a number of suitable visions. The proposed model is based on a class of fractional differential equations. This type of class was a generalized neutral differential equation of first order. Certain sufficient conditions for the existence of periodic outcomes have been imposed.

Key words

Fractional calculus, Fractional differential operator, Fractional differential equation, Psychotherapy

Introduction

One, out of five adults in the world suffers with a diagnosable mental disorder. These disorders are the principal cause of infirmities and remove physical and sensitive toll on these individuals, their families, and their communities. Psychotherapy has been considered as an actual technique for treating these disorders (Lambert, 2002; Muran, *et. al.,* 2009;Memon and Treur, 2010).

The achievement of psychotherapy is governed by the nature of the therapeutic relationship between a therapist and a client. Trainings are required for classifying the most important basics of this relationship. Though, those are basics for not completely unstated, previous psychotherapy trainings have described that the critical part is the personal relationship between the therapist and client, rather than an abstract theoretical framework utilized by the therapist. Preceding trainings of this relationship dyad (something that involves two basics or parts) have used the social science model for defining the functional correlation between dependent and independent variables (Robertson and Combs, 2014; Box *et al.*, 2015).

Fractional order differential equations have been definitely applied on promised in modeling of numerous diverse techniques and arrangements in physics, chemistry, biology, engineering, medicine and food processing. It has shown the effectiveness of this class of differential equations (Ibrahim and Jahangiri, 2014; Ibrahim and Jahangiri, 2015).

The present study aimed to construct a mathematical model skilled in creating hard estimates about the dynamic of psychotherapy with the final purpose of using it to better sequence therapists. The outcomes of the computations reflect a new viewpoint on the therapeutic relationship with suitable visions. The model was based on a class of fractional differential equations. The type of this class is a generalized neutral differential equation of first order. Certain sufficient conditions for the existence of periodic outcomes were imposed.

Materials and Methods

In the present study, certain sufficient conditions were investigated for the existence of periodic outcomes to a fractional Rayleigh-type equation with state-dependent delay of the formula :

$$D^{\wp}(u(t)+\alpha u(t-\beta))=\eta_1(t,u(t))+\eta_2(t,u(t-\gamma))+P(t,u(t)) \quad (10)$$

where, $P : R \to R$ and $\eta_1, \eta_2 : J \times R \to R$, $J = [0, T]$ are continuous functions, α, β and γ are constants, P is T-periodic, η_1 and η_2 are T-periodic in the first argument, $|\alpha| \neq 1$, $T > 0$ and $D^{\wp}$ is the Riemann-Liouville fractional differential operator, which is defined by:

$$D^{\wp}u(t)=\frac{d}{dt}\int_0^t \frac{(t-\tau)^{-\wp}}{\Gamma(1-\wp)}u(\tau)d\tau, \quad \mathrm{T}\in J,\ \wp\in(0,1).$$

While, the Riemann-Liouville definition of the fractional integral of order $\wp$ is formulated as follows:

$$I_t^{\wp}u(t)=\int_0^t \frac{(t-\tau)^{\wp-1}}{\Gamma(\wp)}u(\tau)d\tau, \quad \wp>0.$$

It is to be noted that Equation (1) is a generalization to equations which have been studied by Liu and Huang (2006). Equation (1) describes the relationship between the client $u(t)$ and the therapist $P(t, u(t))$ during time $T \in J$, where this relationship is the emotional valence, or effect of the therapist and client. For instant, a positive value of P would indicate the therapist is in a positive state, and a negative value of P indicates the therapist is in a negative state and similarly for the positive and negative values of u for the client. The functions η_1 and η_2 are the influence functions. They are represented to the activation (change state) and unchanged state or delay in the active case respectively. In addition, they dictate how the two followers of the dyad would influence all other. Though, these influence functions were established on published empirical data, they were slightly projected in nature, but nevertheless could silently offer a preliminary point for this experimental project. Consequently, these influence functions and the empirical basis for their functional formal describe how the therapist's valence depends on the client's valence. It was noted that when the client's move was negative, the therapist displayed more positive disturbance, however, they might under extended contact with the clients negative move, activate to display neutral and even negative disturb in the appearance of dangerous negative behavior. This behavior might be refined during time and take its periodicity values. For this behavior, the boundary value problem was considered $u(0) = u(t) := Tu_0, T \in J$.

When the client was effectively neutral, therapists will normally employ approaches to produce additional positive emotions. Their attempt would encourage clients, or try to develop the client to pay attention on their strengths and abilities, in the confidence that this change of attention would adjust the clients affect. On the other hand, when the therapist was effectively neutral, all the clients were possibly either marginally negative or neutral (chiefly initial in the therapeutic process). The rise of psychoanalysis, at which point clients may respond negatively. It was assumed that the peak of the influence function is as follows:

$$\overline{\eta_1} := sup_{u\in B}|\eta_1(t,u(t))|,\ \overline{\eta_2} := sub_{u\in B}|\eta_2(t,u(t-\gamma))|.$$

By means of the therapist's disturbed moves from neutral to positive, primarily, the client might continue to be neutral or marginally negative. Though, for instance the therapist's disturbance converts more positive, the client may reply positively by displaying more neutral disturb. This might be a signal of the client eitherly obtaining into the therapist communication or a signal that the client is open to experience some positive consequences from the therapeutic intermediation. A positive and steady state might occur at this point, where therapeutic improvements may be maximized. Nevertheless, as a sign of exciting expressions of positive effect on the measure of the therapist, the client's strength turns negative. It is sensible to unsure that this might be a measure of a beginner therapist's practice, but may perhaps likewise be thought of analysts who possibly would be on the edge of breakdown. This obstruction may not even be recognized by the therapist, but then again, it might become selected by the client, and transfer the therapy towards the more negative end of the display.

At the same time, there were circumstances when exhibition of negative sensation may be useful to the therapeutic relationship. Specifically, suitable confrontation or expressions of dissatisfaction might be essentially critical to the client. Moreover, the instantaneous outcome may be a therapeutic disagreement. If it is done firmly or purposefully, it may have an extended term benefit for the client. The achievement of this approach depends a lot on the ability of the therapist and the power of the therapeutic relationship. Finally, each algorithm needs asymptotic, boundedness and convergence. The stability of the solutions is studied in view of the Hadamard well-posed. Therefore, our aim is to establish the existence and the uniqueness of the solution of Equation (1).

Results and Discussion

Our method is based on the fixed point theory. The Banach space is defined as $B := \{u \in C(R, R): u(t+T) = u(t), t$

$\in J := [0, T]\}$, with the maximum norm $\| u \| = \max_{t \in J} | u(t) |$. In the sequel, we assume $\overline{P} := sup_{u \in B} | P(u(t)) |$. We have the following finding:

Theorem 3.1 : Consider Equation (1). If $|\alpha| + T < 1$, then it has at least one solution in B. Moreover, if, $\alpha < 1$, then it is T-periodic solution.

Proof : Equation (1) can be converted in the integral form as follows:

$$u(t) = Tu_0 - \alpha u(t-\beta) + \int_0^t \frac{(t-\tau)^{\wp-1}}{\Gamma(\wp)}) \\ [\eta_1(\tau, u(\tau)) + \eta_2(\tau, u(\tau-\gamma)) + P(\tau, u(\tau))]d\tau.$$

Define an operator Q: B $\rightarrow$ B by

$$Qu(t) = Tu_0 - \alpha u(t-\beta) + \int_0^t \frac{(t-\tau)^{\wp-1}}{\Gamma(\wp)}) \\ [\eta_1(\tau, u(\tau)) + \eta_2(\tau, u(\tau-\gamma)) + P(\tau, u(\tau))]d\tau.$$

Thus, we have

$$|Qu(t)| \leq T|u_0| + |\alpha|\,|u(t-\beta)| + \int_0^t \frac{(t-\tau)^{\wp-1}}{\Gamma(\wp)}) \\ |\eta_1(\tau, u(\tau)) + \eta_2(\tau, u(\tau-\gamma)) + P(\tau, u(\tau))|d\tau \\ \leq T|u_0| + |\alpha|\,|u(t-\beta)| + \frac{T^{\wp}}{\Gamma(\wp+1)}[\overline{\eta}_1 + \overline{\eta}_2 + \overline{P}].$$

Hence, Q is bounded and continuous in B. Taking the maximum norm, we obtain

$$\| u \| (1-|\alpha|) \leq T|u_0| + \frac{T^{\wp}}{\Gamma(\wp+1)}[\overline{\eta}_1 + \overline{\eta}_2 + \overline{P}].$$

For positivity respond of the client, we have $| u_0 | \leq \| u \|$, then we conclude that

$$\| u \| (1-|\alpha|) \leq T|u_0| + \frac{T^{\wp}}{\Gamma(\wp+1)}[\overline{\eta}_1 + \overline{\eta}_2 + \overline{P}] \\ \leq T \| u \| + \frac{T^{\wp}}{\Gamma(\wp+1)}[\overline{\eta}_1 + \overline{\eta}_2 + \overline{P}].$$

Hence, we attain

$$\| u \| \leq \frac{\frac{T^{\wp}}{\Gamma(\wp+1)}[\overline{\eta}_1 + \overline{\eta}_2 + \overline{P}]}{1-|\alpha|-T}.$$

It is clear that the set is

$$S := \{u : \| u \| \leq r = \frac{\frac{T^{\wp}}{\Gamma(\wp+1)}[\overline{\eta}_1 + \overline{\eta}_2 + \overline{P}]}{1-|\alpha|-T}\}$$

It is non-empty, compact and convex set and Q is compact. In view of the Schauder fixed point theorem, the operator Q has at least one fixed point in S, which corresponds to the solution of Equation (1). The periodicity of the solution comes from the fact that all the functions in Equation (1) are T-periodic, we have

$$\int_0^T u(t)dt = \frac{Tu_0}{1-\alpha},$$

but u_0 is arbitrary, thus u is a T-periodic solution.

Conversely, for the negative response of the client, we have $| u_0 | \geq \| u \|$ that is the problem in ill-Hadamard case and there is no stable solution in the set S. In this situation, the psychotherapy is faulty and the nature of the relationship between a therapist and a client is unsuccessful. This completes the proof.

We proceed to apply the idea of the Hadamard well-posed, by studying the uniqueness of the solution. A function $f(w)$ is achieved in the Lipchitz condition of order $0 < \wp \leq 1$ if and only if $|f(w) - f(v)| \leq \in | w - v |$, for some constants $\in \geq 0$. We have the following result:

Theorem 3.2 : Consider Equation (1). If all the functions η_1, η_2 and P are achieved the Lipchitz condition in the second variable with the constants $k_1 > 0$, $k_2 > 0$ and $k_3 > 0$ respectively, then it has a unique solution when

$$(|\alpha| + \kappa_1 + \kappa_2 + \kappa_3) < \frac{\Gamma(\wp+1)}{T^{\wp}}.$$

Proof : Assume the operator Q in Theorem 3.1. Let u and v be in the space $S \subset$ B. Thus, we obtain

$$|Qu(t) - Qv(t)| \leq |\alpha|\,(|u(t-\beta) - v(t-\beta)|) \\ + \int_0^t \frac{(t-\tau)^{\wp-1}}{\Gamma(\wp)})(|\eta_1(\tau, u(\tau)) - \eta_1(\tau, v(\tau))| \\ +|\eta_2(\tau, u(\tau-\gamma)) - \eta_2(\tau, v(\tau-\gamma))| + |P(\tau, u(\tau)) \\ - P(\tau, v(\tau))|)d\tau \leq |\alpha|\,(|u(t-\beta) - v(t-\beta)|) \\ + \frac{T^{\wp}}{\Gamma(\wp+1)}(\kappa_1 + \kappa_2 + \kappa_3)|u-v| \\ \leq (|\alpha| + \kappa_1 + \kappa_2 + \kappa_3)\frac{T^{\wp}}{\Gamma(\wp+1)}|u-v| \\ := \lambda|u-v|.$$

Since $0 < \lambda < 1$, then Q is a contraction mapping. Hence, by virtue of the Banach fixed point theorem, it can be concluded that Q has a unique fixed point in S, which

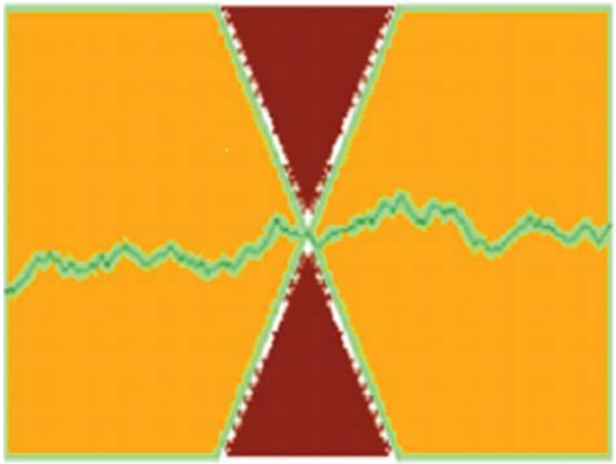

Fig. 1 : The behavior of the Lipchitz function (client behavior)

corresponds to the solution of (1). This completes the proof.

The present investigation is definitely an important theoretical approach to develop a new method that may shed light on the dynamics of psychotherapy. Certainly, it will serve as a stable starting point to further new theoretical and experimental studies. Theoretically, it is established that variations in the influence functions with time were necessary to establish how relationships in marriages were improved (or worsened). We wish to discover how changing these influence functions, during the time and the case of the client, can progress the therapeutic outcome. We are especially interested in learning how the influence functions are dynamically changed between innocent and skilled therapists. Fig. 1 shows that if the function is Lipchitz then the graph of the function everywhere lies completely outside this cone (unstable area). That means the client gets a good relationship with the therapist.

As evident from the discussion, this mathematical model is based on the neutral differential equation. It is almost intended to represent the full nature of the complex human communication in psychotherapy. Nonetheless, the information is significant in understandings therapist neutrality, client expressive fluctuates, the mutual roles of inertia and influence between therapist and client. It may achieve some simple dynamical features that may motivate more complex behaviors, emerging in the therapeutic relationship. The uniqueness implies that a client who is less influenced by their own previous state follows an analogous outcome to one that is more approachable to their therapist.

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References

Box, S.J., B. Copley, J. Magagna and E. Moustaki: Psychotherapy with families: An analytic approach. Routledge (2015).

Ibrahim, R.W. and J.M. Jahangiri: Boundary fractional differential equation in a complex domain. *Boun. Val. Probl.*, **66**, 1-13 (2014).

Ibrahim, R.W. and J.M. Jahangiri: Existence and uniqueness of an attractive nonlinear diffusion system. *App. Mathe. Comput.*, **257**, 169-177 (2015).

Lambert, M.J. and D.E. Barley: Research summary on the therapeutic relationship and psychotherapy outcomes. In: Psychotherapy relationships that work: therapist contributions and responsiveness to patient needs (Ed.: J.C. Norcross). Oxford University Press, New York (2012).

Liu, B. and L. Huang: Existence and uniqueness of periodic solutions for a kind of first order neutral functional differential equations. *J. Math. Anal. Appl.*, **322**, 121-132 (2006).

Memon, Z.A. and J. Treur: On the reciprocal interaction between believing and feeling: an adaptive agent modelling perspective. *Cogn. Neurodyn.*, **4**, 377-394 (2010).

Muran, J.C., J. D. Safran, B. S. Gorman, L. W. Samstag, C. E. Carter and A. Winston: The relationship of early alliance ruptures and their resolution to process and outcome in three time-limited psychotherapies for personality disorders. *Psychother. Theo. Res.*, **46**, 233-248 (2009).

Robertson, R. and A. Combs : Chaos theory in psychology and the life sciences, Psychology Press, Taylor and Francis Group, New York and London (2014).