

Generalized height-diameter models for *Picea orientalis* L.

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Abstract: The purpose of this paper is to compare major models for height estimation of *Picea orientalis* L trees based on the individual tree diameter and certain other stand variables. The data were collected from 440 trees of pure and even-aged *P. orientalis* stands that are located near Artvin in the northeastern part of Turkey. The data from 406 trees were used for model development and the remaining data were reserved for model validation. A total of 17 non-linear models were fitted to 406 trees. Mean square errors and R^2 values for the 17 models showed that some models reduced similar height estimation. The model [8] gave the best height estimates for *P. orientalis* with the highest R^2 (0.8703) and the lowest mean square error (5.47). Validation of the models using independent data sets showed that model [8] and [16] gave the best height predictions for this particular dataset.

Key words: Generalized diameter-height equations, Model evaluation and validation, Oriental spruce
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Introduction

Individual-tree heights and diameters are essential forest inventory measurements for estimating timber volume and site index, and are also important variables in growth and yield modeling. Forest resource managers require tree volume information to produce yield estimates for timber inventory and improve forest management decision-making (Peng, 1999).

The relationship between tree heights and diameters is one of the most important elements of forest structure. Accurate height-diameter models are required for estimating individual tree volume and site index as well as for describing stand growth dynamics and succession over time (Curtis, 1967; Botkin *et al.*, 1972). Measuring tree height is costly however, and foresters usually welcome an opportunity to estimate this variable with an acceptable accuracy. Missing heights may be estimated using a suitable height-diameter functions (Temesgen and Gadow, 2004). A number of height-diameter equations or models have been developed for various tree species in the world. These models can be fitted to linear or non-linear functions. For example, Huang *et al.* (1992) selected and compared 20 non-linear height-diameter functions, fitted by weighted nonlinear least-squares regression for white spruce (*Picea glauca* Voss.) and aspen (*Populus tremuloides* Michx.). Fang and Bailey (1998) investigated 33 height-diameter equations, including S-shaped and concave-shaped curves, for tropical forests on Hainan Island in southern China. Sanchez *et al.* (2003) estimated 26 linear and non-linear height-diameter functions for *Pinus radiata* throughout Galicia in the northwest of Spain. However, in Turkey, there have been no models or equations developed for many species (including *P. orientalis*) to estimate individual tree height using tree diameter.

Thus, in forest inventory one- or two-entry volume tables are commonly used to estimate tree or stand volume. However, oriental spruce is one of the main timber species of northern Turkey (Tufekcioglu *et al.*, 2004; Akkuzu and Guner, 2008).

In practice, tree height-diameter equations can be used to predict the "missing" heights from field measurement of tree diameters (Larsen and Hann, 1987), and to estimate individual tree biomass using appropriate single-tree biomass equations (Singh, 1982; Penner *et al.*, 1997). In a forest inventory, total tree height is often estimated from observed tree diameter at breast height outside bark. Tree diameter can easily be measured at low cost. But tree height data are relatively more difficult and costly to collect. Thus, models based solely on diameter measurements are most cost effective (Peng, 1999).

The aim of the present study, through the application and comparison of the existing models, is to identify an equation that can be used to predict the individual tree height in pure and even-aged *Picea orientalis* L. stands in northeastern Turkey by considering tree diameter and a number of stand variables, such as dominant diameter, dominant height, age and density.

Materials and Methods

The study area was in the Artvin Forest Planning Unit surrounding the city of Artvin in the Eastern Black Sea Region of Turkey and characterized by a dominantly steep and rough terrain with an average slope of 62% (lat. 41°15' N, long. 41°45' E, alt. 400-2220 m above sea level). Winters are mild and wet, and summers are relatively cool and dry. Mean annual temperature of the study area is 11.9°C, and mean annual precipitation is 719 mm. Main soil types are sandy clay loam, clay loam and sandy loam (Gunlu, 2003).

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Table - 1: Characteristics of the tree samples used for model fitting

Sample for model fitting (N=406)				
Variable	Mean	Maximum	Minimum	SD
Height	20.85	38.80	4.20	6.48
Diameter	34.12	82.00	8.10	12.80

Total height and diameter at breast height (DBH) data were collected from 440 standing trees (406 of 440 fitting and 34 of 440 validation) in 110 circular temporary plots randomly selected from fifteen pure, even-aged stands of *P. orientalis*. Plot sizes were 400-800 square meters depending of stand crown closures. The DBH over bark of all of the trees in each plot was crosswise measured, using calipers, to the nearest millimeter. Dominant heights and ages were measured in 3-5 trees in each plot. Heights were measured, using a Blume-Leiss hypsometer, to the nearest 0.1 meter, and ages were determined using increment borers.

Stand variables calculated from the data collected in the inventories included basal area, quadratic mean diameter, maximum diameter, dominant diameter, mean height and dominant height. The mean, maximum and minimum values and standard deviations of the main dendrometric and stand variables were given in Table 1,2 respectively.

A great number of height-diameter models have been reported in the forestry literature, many of which have been developed for a particular species or specific area. For this study, a total of 17 generalized non-linear height-diameter models were selected for *Picea orientalis*. Some of these models were original, and others were derived.

The terminology used in the description of the models is as follows: h = total height of tree, in m; d = diameter at breast height over bark, in cm; G = basal area of the stand, in $m^2 ha^{-1}$; d_g = quadratic mean diameter of the stand (cm); D_{max} = maximum diameter of the stand (cm); D_0 = dominant diameter of the stand (cm); H_0 = dominant height of the stand (m); H_m = mean height of the stand (m); t = age of the stand (years m); N = number of trees per hectare; \log = logarithm 10; n = natural logarithm; e = base of the natural logarithm (≈ 2.71828); b_i = regression coefficients to be determined by model fitting.

The data from 406 standing trees were used to estimate the parameters of each model in Table 3. Tree data for 34

individual trees were used for validation purposes. In order to estimate the parameters of all models and validate the models, SPSS statistical program was used. The SPSS Regression-Nonlinear procedure was used to estimate parameters in nonlinear regression models.

Comparison of the model estimates was based on graphical and numerical analysis of the residuals and values of two statistics: the mean square error (MSE), which analyses the precision of the estimates; and the adjusted coefficient of determination (R^2_{adj}), which reflects the part of the total variance that is explained by the model. The expressions for these statistics are as follows:

$$\text{Mean square error: } MSE = \frac{\sum_{i=1}^N (H_i - \hat{H}_i)^2}{N - p}; \quad (1)$$

$$\text{Coefficient of determination: } R^2 = 1 - \frac{\sum_{i=1}^N (H_i - \hat{H}_i)^2}{\sum_{i=1}^N (H_i - \bar{H}_i)^2}; \quad (2)$$

Adjusted coefficient of determination:

$$R^2_{adj} = 1 - \left(1 - R^2\right) \times \left(\frac{N - 1}{N - p}\right); \quad (3)$$

where H_i is the observed height for the i th tree, \hat{H}_i is the predicted height for the i th tree, \bar{H}_i is average of observed height for the i th tree, \bar{H}_i/N is the number of observation, p is the number of parameters to estimate and R^2 is the coefficient of determination.

Results and Discussion

In this study height models for *P. orientalis* were estimated for practical use in Artvin. All the models were statistically significant ($R^2 > 70$, $p < 0.05$). They can be applied to predict individual tree height using tree and stand characteristics.

Since these models were tested for diameters between 8.1 cm and 82 cm, and for heights between 4.2 m and 38.8 m, estimation of height of a tree may not be exactly true for the minimum values; in fact, further research on height prediction for young trees is needed.

Table - 2: Characteristics of the plots from which the samples of trees used for model fitting

Sample for model fitting (N=406)				
Variable	Mean	Maximum	Minimum	SD
G	44.60	76.50	16.53	13.56
d_g	29.30	49.51	13.56	7.94
D_{max}	52.10	90.40	22.00	11.63
D_0	44.94	62.73	21.70	7.75
H_0	23.31	32.65	12.05	4.51
H_m	20.90	31.62	11.10	4.40
t	88.47	134.00	37.00	21.12
N	769.14	1950.00	187.50	370.51

Table - 3: Generalized height-diameter models selected

Model no.	References	Expression
1	Curtis (1970)	$h = 10 \left(b_0 + b_1 \frac{1}{d} + b_2 \frac{1}{t} + b_3 \frac{1}{d_g t} \right)$
2	Cox I (1994)	$h = e^{(b_0 + b_1 \ln d_g + b_2 \ln N + b_3 \sqrt{d})}$
3	Clutter and Allison (1974)	$h = 1.3 + 10 \left(b_0 + b_1 \frac{1}{d} + b_2 \frac{1}{\sqrt{t}} + b_3 \frac{1}{d\sqrt{t}} + b_4 \frac{\log N}{\sqrt{t}} \right)$
4	Cañadas <i>et al.</i> I (1999)	$h = 1.3 + (H_0 - 1.3) \left(\frac{d}{D_0} \right)^{b_0}$
5	Gaffrey (1988)**	$h = (H_0 - b_0) e^{b_1 \left(1 - \frac{d_g}{d} \right) + b_2 \left(\frac{1}{d_g} - \frac{1}{d} \right)}$
6	Sloboda <i>et al.</i> (1993)	$h = 1.3 + (H_m - 1.3) e^{b_0 \left(1 - \frac{d}{d_g} \right)} e^{b_1 \left(\frac{d}{d_g} - \frac{1}{d} \right)}$
7	Harrison <i>et al.</i> (1986)	$h = H_0 \left(1 + b_0 e^{b_1 H_0} \right) \left(1 - e^{-\frac{b_2 d}{H_0}} \right)$
8	Pienaar <i>et al.</i> (1990)**	$h = b_0 + H_0 * \left(1 - e^{\left(\frac{-b_1 * d}{\ln D_0} \right)} \right)^{b_2}$
9	Hui and Gadow (1993)	$h = 1.3 + b_0 H_0^{b_1} d^{b_2 H_0^{b_3}}$
10	Mirkovich (1958)	$h = 1.3 + (b_0 + b_1 H_0 - b_2 d) e^{\left(\frac{-b_3}{d} \right)}$
11	Cox III (1994)*	$h = H_m \left(b_0 + b_1 H_m + b_2 \frac{H_m}{d_g} + b_3 d + b_4 \frac{N}{dg(H_m d_g)} \right) \frac{1}{d}$
12	Schröder and Alvarez II (2001)*	$h = 1.3 + (b_0 + b_1 H_0 - b_2 d_g + b_3 G) e^{\left(\frac{-b_4}{\sqrt{d}} \right)}$
13	Cox II (1994)*	$h = b_0 + b_1 H_m + b_2 d_g^{0.95} + b_3 e^{-0.08d} + b_4 H_m^3 e^{-0.08d} + b_5 d_g^3 e^{-0.08d}$
14	Tomé (1989)	$h = H_0 e^{\left(b_0 + b_1 H_0 + b_2 \frac{N}{1000} + b_3 * t \right) \left(\frac{1}{d} - \frac{1}{D_0} \right)}$

15 Lenhart (1968)
$$h = \frac{H_0}{e^{b_0 + \left(\frac{1}{d} - \frac{1}{D_{max}}\right) \left(b_1 + b_2 \ln N + b_3 \frac{1}{t} + b_4 \ln H_0 \right)}}$$

16 Amateis et al. (1995)
$$h = b_0 H_0^{b_1} 10^{\left(\frac{b_2}{t} + \left[\frac{1}{d} - \frac{1}{D_{max}} \right] \left[b_3 + b_4 \frac{\text{Log}N}{t} \right] \right)}$$

17 Pascoa (1987)
$$h = b_0 H_0^{b_1} G^{b_2} N^{b_3} e^{\left(\frac{b_4 + b_5}{t} \right)}$$

* Modifications of the original model by Sanchez et al, 2003; ** Modifications of the original model in this study

Table - 4: Values of the statistics for fitting and validation models

Model No.	Fitting data (N=406)			Validation data (N=34)		
	R ²	R ² _{adj}	MSE	R ²	R ² _{adj}	MSE
1	0.7560	0.7542	10.3235	0.8653	0.8519	7.1615
2	0.7313	0.7293	11.3696	0.7460	0.7206	13.5111
3	0.7330	0.7303	11.3267	0.8381	0.8158	8.9065
4	0.8016	0.8016	8.3330	0.8274	0.8274	8.3460
5	0.7838	0.7832	9.1033	0.8427	0.8378	7.8409
6	0.7305	0.7298	11.3464	0.6597	0.6491	16.9653
7	0.8624	0.8617	5.8073	0.8709	0.8626	6.6455
8	0.8703	0.8697	5.4712	0.9043	0.8981	4.9259
9	0.8290	0.8277	7.2367	0.8176	0.7993	9.7024
10	0.8684	0.8674	5.5665	0.9037	0.8940	5.1231
11	0.7823	0.7801	9.2351	0.7041	0.6633	16.2799
12	0.8561	0.8547	6.1021	0.8805	0.8640	6.5753
13	0.8448	0.8429	6.5992	0.8729	0.8502	7.2433
14	0.8593	0.8583	5.9505	0.8804	0.8684	6.3605
15	0.8626	0.8613	5.8267	0.8989	0.8850	5.5603
16	0.8506	0.8491	6.3362	0.9167	0.9053	4.5804
17	0.8676	0.8660	5.6283	0.9035	0.8862	5.5009

Table - 5: Parameter values of the selected models

Model No.	Parameters					
	b ₀	b ₁	b ₂	b ₃	b ₄	b ₅
1	1.5909	-7.3055	4.3883	-194.7479		
2	-0.9334	0.4928	0.1785	0.1974		
3	1.6935	-9.2366	-0.7383	3.3951	-0.0488	
4	0.5139					
5	3.0347	0.5674	-0.9785			
6	-0.4686	-0.0801				
7	0.1273	0.0285	-0.9326			
8	3.4184	0.2646	2.3351			
9	4.9729	-0.1867	0.1434	0.4296		
10	9.9540	1.0706	0.0422	16.8093		
11	0.6893	-0.0237	0.4434	0.0148	-8.7442	
12	20.2083	1.8328	0.1075	0.0229	6.4383	
13	7.3317	0.9943	-0.1206	-22.0233	-0.0017	-0.00026
14	-2.8268	-0.5812	-9.0798	0.0419		
15	-0.0758	-42.1121	1.0774	64.1099	15.5945	
16	1.6379	0.8600	1.1177	-5.7799	-22.7094	
17	3.9152	0.6918	-0.0232	0.0199	-3.2477	-15.5396



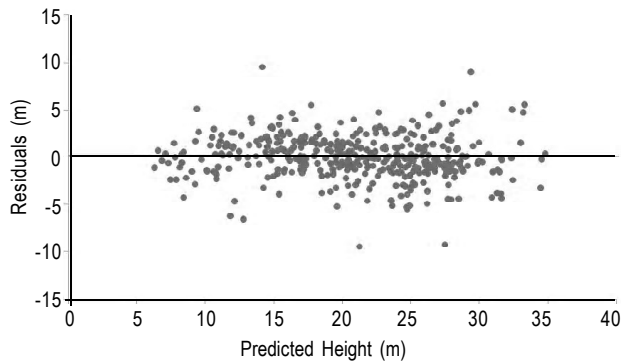


Fig. 1: Plot of residuals versus predicted values in the fitting phase for the model [8]

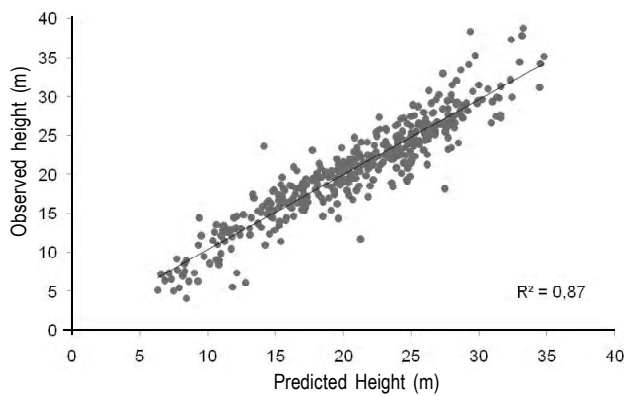


Fig. 2: Plot of observed values versus predicted values in the fitting phase for model [8]

In Table 3, most of the functions performed well in describing the height estimates for fitting and testing data for *P. orientalis* in Artvin. The values of the statistics used to compare the models in the fitting and validation phase are given in Table 4. The models [1], [2], [6] and [11] produced poor height estimates. In addition, these models had the highest MSE values. In Table 5, the parameter values of the analyzed models were listed.

In the fitting phase, the models [2], [3] and [6] produced poor height estimates. Several models for fitting data perform well and produce very similar results. The model [8] gives the best performance according to the values of the statistics used to compare the models in the fitting phase.

Plot of residuals versus the heights predicted in the fitting phase of the model [8] are shown in Fig. 1. This figure supports the hypothesis of normality, homogeneity of variance and independence of residuals.

Plot of the observed heights versus the predicted heights is also drawn for fitting data. The plot shows that the model [8] fits the data well (Fig. 2) since R^2 is 0.87 and MSE 5.47.

The models [2], [6] and [11] appear to result in relatively weak estimation of tree height in the testing phase. The model [16] gave the best performance according to the values of the statistics

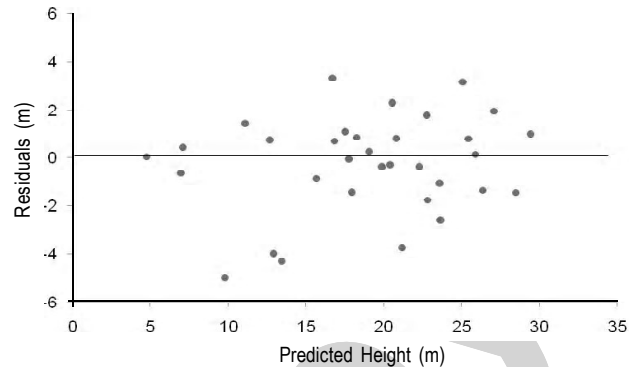


Fig. 3: Plot of residuals versus predicted values in the testing phase for the model [16]

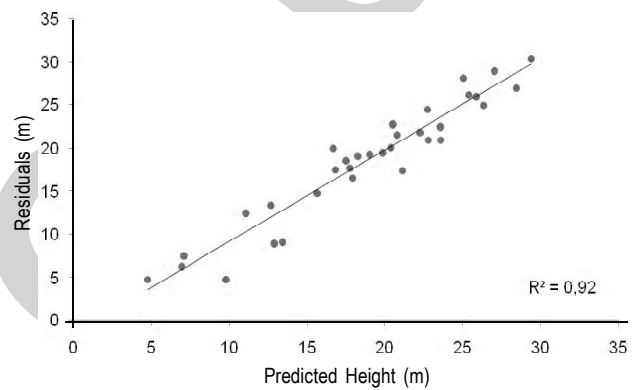


Fig. 4: Plot of observed versus predicted values in the testing phase for model [16]

used to compare the models in the testing phase. Plot of residuals versus the heights predicted in the testing phase of the model [16] is shown in Fig. 3.

Plot of the observed heights versus the predicted heights is also drawn for testing data. The plot shows that the model [16] fits the data well (Figure 4) since R^2 is 0.92 and MSE 4.58. A simple linear model, Actual Height = $a + b \cdot$ Predicted Height, was estimated on the testing data in Fig. 4. Estimated coefficients from this simple linear fit are: $a = -1.376$ and $b = 1.059$ ($p < 0.05$).

The model [16] which uses diameter, dominant diameter, dominant height, age and number of trees per hectare is the best model to predict the height for testing data. But the measurements of these stand characteristics are difficult and time consuming according to the model [8]. If the dominant diameter and dominant height of the stand, and tree diameter were measured, the model [8] can be recommended for *P. orientalis*. It is not so difficult and time consuming that these measurements are done.

The inclusion of the mean height as an independent variable in the height-diameter equations appears to be necessary in order to achieve acceptable predictions. This requires the measurement of at least one sample of heights for the practical application of the equation. The best predictions of height for testing data are obtained by the model [16] which depends on five parameters (d , d_p , H_p , t

and N), but the model [8] gives the best predictions of height for fitting data which uses fewer parameters (d , d_0 and H_0).

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